Dynamic Branching Adjustment Complexity Proof

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1 Theorem: Dynamic Branching Adjustment Complexity

Statement: Adjusting branching factor in an N-way splay tree takes $O(1)$ amortized time per operation.

2 Proof

1. Branching adjustment triggered when: $|\text{children}| > \text{maxChildren}$ or optimal branching changes

2. Split operation: $O(t)$ where t is current branching factor

3. Since $t \leq \sqrt{n}$ (by design), split cost: $O(\sqrt{n})$

4. Split occurs at most once per $O(\sqrt{n})$ insertions

5. Amortized cost: $\frac{O(\sqrt{n})}{O(\sqrt{n})} = O(1)$

Conclusion: Dynamic branching adjustment has $O(1)$ amortized time complexity.